

Homework 1

PHYS798C Spring 2024

Due Thursday, February 8, 2024

Homework Policy:

Your grade will be based on homework and class participation. In exchange for not giving exams, I ask that you do the homework. You may work on homework together, but not doing the homework will imperil your grade—I am willing to give bad grades if homework is not done. Please hand in your homework on time. I will not accept late homework, unless a valid excuse (such as illness) is given, preferably before the homework is due.

1. Bulk London Screening.

(a) Use the second London equation, and Maxwell's equations (ignoring displacement current), to show that

$\vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = -\frac{1}{\lambda^2} \vec{B}$ in a superconductor, where λ is the magnetic penetration depth.

(b) Suppose a semi-infinite superconductor has a surface that lies in the $y-z$ plane. A magnetic field is applied in the z direction parallel to the surface, $\vec{B} = (0, 0, B_0)$. Given that inside the superconductor the magnetic field is a function of x only, $\vec{B} = (0, 0, B_z(x))$ show that

$$\frac{d^2 B_z(x)}{dx^2} = \frac{1}{\lambda^2} B_z(x).$$

(c) Solving the ordinary differential equation in (b) show that the magnetic field near a surface of a superconductor has the form $B_z(x) = B_{z0} e^{-x/\lambda}$.

(d) Find the current distribution $\vec{J}_s(x)$ inside the superconductor.

2. Screening in a superconducting slab. Solve the screening equation from Problem 1 for an infinite superconducting plate of finite thickness $2t$, assuming the magnetic field B_0 is applied parallel to *both* surfaces. Find simple analytical expressions for both the magnetic field and the supercurrent inside the slab. As examples, plot the current and magnetic field for a thickness $2t = \lambda$, and $2t = 10\lambda$. *Hint: put the origin of the coordinate system in the center of the slab.*

3. Two-fluid model. A more realistic model for a superconductor assumes that there is a density n_n of normal electrons which obey a Drude-like equation,

$$\frac{dJ_n}{dt} = \frac{n_n q^2}{m} E - \frac{J_n}{\tau}$$

as well as a density n_s of superelectrons which obey a London equation,

$$\frac{dJ_s}{dt} = \frac{n_s q^2}{m} E.$$

(a) Using the $e^{+i\omega t}$ time convention, find the frequency-dependent complex conductivity $\sigma(\omega) = \sigma_1 - i\sigma_2$. Assume that each 'fluid' responds independently to the electric field, so therefore their contributions add up to form the total conductivity.

(b) What simple lumped-element circuit has an admittance $Y = 1/Z$ (where Z is the complex impedance of the circuit) with the same frequency dependence as $\sigma(\omega)$?

(c) Show that, in the low-frequency limit ($\omega\tau \ll 1$), the normal-fluid response is purely ohmic, while the superfluid response is purely inductive. In this limit, plot $\sigma_1(T)$ and $\sigma_2(T)$ vs T using the empirical relationships:

$$n_s(T) = n_0 [1 - (T/T_c)^4]; n_n(T) = n_0 - n_s(T),$$

where n_o is the density of electrons in the material.

Aside: The expression for $n_s(T)$ is a fairly good approximation for the superfluid density in a clean metal, but the second expression is seriously flawed: $n_s(T) + n_n(T)$ is not equal to the total electron density.

4. **Meissner screening in COMSOL.** Here is a numerical tool that allows us to study static and dynamic properties of superconductors in a convenient manner. In this problem we will solve for the static Meissner screening of a superconducting cylindrical disk in a dc magnetic field using the London equations. We will use COMSOL to solve this problem and visualize the results. Follow the step by step instructions in this book (available to all UMD students at: [Shortcut to Superconductivity](#)) to reproduce this 3D image of the superconducting disk in the Meissner state, Figure 1.

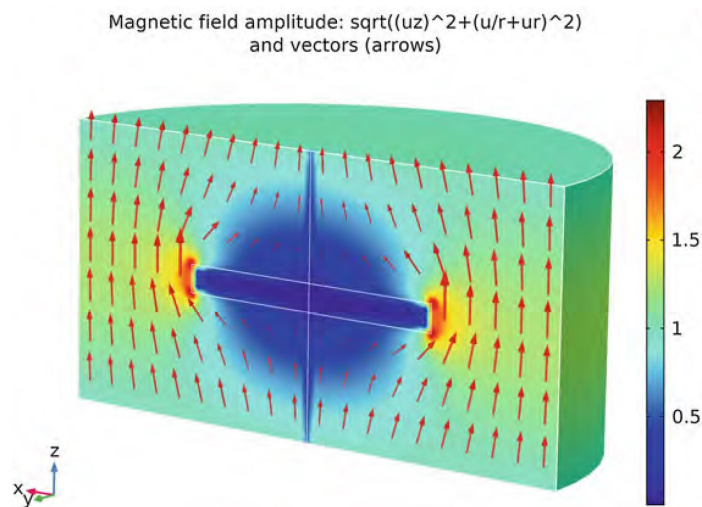


Fig. 1.11 Meissner effect in London's approximation visualized via COMSOL

Figure 1: Magnetic field around a superconducting disk (horizontal blue rectangle at center) in the Meissner state. From Armen Gulian, "Shortcut to Superconductivity", (Springer International Publishing, 2020).

a) Reproduce the figure above based on the instructions in the book starting on page 10. There are detailed step-step instructions also available in the Homework 1 folder under the Assignments tab on ELMS.

b) One distracting feature of the plot is the blue spike in the $|\vec{B}|$ color plot extending vertically along the cylindrical axis of the disk, which is evident in the figure above. The origin of this feature may be that COMSOL has issues with $\frac{0}{0}$ situations. Correct this problem and replot the magnetic field profile, in the same format as the figure above.

Other resources:

You can run COMSOL on the university's Virtual Computer Lab, available here: UMD VCL. You may have to install Citrix Workspace on your computer to access COMSOL through the VCL. Note that installing Citrix on your computer may require that you run "Make Me Admin" to establish Administrator privileges. In addition, you may have to right click on the downloaded Citrix .exe file and choose "Run as Administrator" to complete the installation.

Here are some papers that utilize COMSOL to study superconductivity:

- 1) J.-G. Caputo, L. Gozzelino, F. Laviano, G. Ghigo, R. Gerbaldo, J. Noudem, Y. Thimont, and P. Bernstein, "Screening magnetic fields by superconductors: A simple model," *J Appl Phys* 114 (23), 233913 (2013).
- 2) Bakhrom Oripov and Steven M. Anlage, "Time-dependent Ginzburg-Landau treatment of rf magnetic vortices in superconductors: Vortex semiloops in a spatially nonuniform magnetic field," *Phys Rev E* 101 (3), 033306 (2020).